First Steps to Math Success

Guidance for Teaching Mathematics in Transitional Kindergarten

DEVELOPED BY THE
Curricular and Improvement Support Committee
CISC Mathematics Subcommittee
Transitional Kindergarten Workgroup
First Steps to Math Success: Guidance for Teaching Mathematics in Transitional Kindergarten was developed by a workgroup as part of the CISC Mathematics Subcommittee. Primary authors of the guidance document, Mark Alcorn, Jacqueline Booker, Jon Dueck, Dennis Regus, Kirsten Sarginger, and Van Lay, are passionate about helping our youngest students succeed in mathematics. The workgroup acknowledges the California Early Math Initiative, the State Board of Education staff, the CISC Math Subcommittee, members of the County Office Early Care and Education programs, and CISC Executive Committee for their guidance in creating this document. The workgroup acknowledges the support of the California County Superintendents.

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As districts and schools plan to expand Universal Prekindergarten, creating the conditions to support joyful, robust foundational learning in mathematics will be a primary focus. That said, leaders and partners must be mindful that learning and discovering can change dramatically as children develop from ages 48 to 72 months. Therefore, the Prekindergarten math program should reflect and respond to learners' developmental stages. Some guiding questions for responsive planning might include:

- What is important to experience and learn about?
- What does math look like when you are 4 years old?
- What does it mean to teach students who are 48 months old?
- What does the teacher need to be aware of?
- Where do free choice and play fit in?
- What manipulatives/resources will be needed in the classroom?
- How can the Preschool Learning Foundations for Mathematics offer guidance?

Many 4-year-old children currently attend school. With the expansion of Universal Prekindergarten, California will increase the number of 4-year-olds in the elementary school setting and potentially decrease those in preschool. This document describes the components of a strong mathematical classroom for our youngest learners.

**BACKGROUND**

- Universal Transitional Kindergarten is one element of Universal Pre-Kindergarten. Other programs such as CA State Preschool Programs and Head Start are part of the offerings for providing universal access to high quality pre-K education for every 4 year old whose family wants it. This is free for those who qualify.
- Transitional Kindergarten is currently available in the 2022-2023 school year for students who turn 5 years old between September 2 and February 2.
- Assembly Bill (AB) 130 phases in eligibility for Transitional Kindergarten starting in the 2022-23 school year through 2025-26 school year by expanding the age eligibility by 2 months each year. By 2025-26, all children who turn 4-years-old by September 1st can be enrolled in TK.
MINDSET

Developing a growth mindset in our teachers is essential to the long term success of our youngest learners. The Teaching for Robust Understanding (TRU) Framework describes 5 dimensions of a powerful math classroom. One of these dimensions, *Agency, Ownership, and Identity*, directly impacts the concept of mindset in the classroom. Throughout a child’s UPK experience, they should continually be provided with opportunities to develop their sense of agency, to productively struggle with math concepts, and to begin to gain a sense that they are capable mathematicians.

Helping students develop a positive mathematical disposition is a key focus of early mathematics. Many teachers report having less than positive experiences with mathematics in their own schooling. To this end, mathematical experiences in UPK must be grounded in play and exploration. There should be a wide variety of resources, such as manipulatives, in the classroom to encourage hands-on exploration. Mathematics should be fun. Positive mathematical identity is crucial not only for children, but for the adults who support them as well. This TED Talk by Dan Finkel is an excellent place for teachers to start their journey.

IMPLICATIONS FOR THE FUTURE

Even though young children acquire some arithmetic skills naturally through their interactions with the world around them, they benefit greatly from high-quality math instruction before they enter kindergarten. Children who enter kindergarten with strong math knowledge have higher math and reading achievement at the end of elementary school than children who enter kindergarten with weak math skills. Young children learn math best through a combination of play-based, everyday experiences as well as structured, age appropriate learning activities.

Mounting evidence indicates that the math knowledge children develop before entering elementary school is critical to later academic achievement. In a widely-cited study of longitudinal data, Duncan et al, (2007) report that in a comparison of math, literacy, and social-emotional development at kindergarten entry, “early math concepts such as knowledge of numbers and ordinality were the most powerful predictors of later learning.” Indeed, research consistently indicates that early mathematical proficiency is associated with later proficiency not only in mathematics, but in reading as well (see Volume 46, Issue 5, of Developmental Psychology; Eccles, 2010) and may even be linked to rates of high school graduation. Although the mechanisms underlying such associations are not yet understood, the importance of early math, and of access to it for all students, is clear (*Math Matters: Children’s Mathematical Journey Start Early*).
Children who begin school with poor math skills typically do not catch up. Across the nation children who have low math scores in the fall of their kindergarten year continue to lag behind their better prepared peers through the 8th grade. Those least prepared are disproportionately children of color and from low-income families. Clearly, any serious effort to close the achievement gap needs to include, if not focus on, children before school entry. (Math Matters: Children's Mathematical Journey Start Early)

**CONTENT**

A key element of the PreKindergarten (PK) experience in preparing for the Kindergarten California Common Core State Standards: Mathematics (CA CCSSM) is understanding the content of the Preschool Learning Foundations. By anchoring UPK Math instruction on the Preschool Foundations and Frameworks, we can develop a common understanding of these learning progressions, especially as students in TK become increasingly younger and the progressions are expanded to 66 months. To support teachers in meeting students at their individual level of developmental readiness, the California Department of Education has created an alignment document between the current Preschool Learning Foundations and the Kindergarten CA CCSSM. Transitional Kindergarten teachers can use this progression to formatively assess each student's current levels of understanding and provide learning opportunities that extend into the next levels of learning.

**Developmentally-informed practices should be the driving force behind content choices and instruction as it takes into consideration a child's age, interest, abilities and socio-cultural context.**

Readiness needs to be realized from a developmental perspective. Individual interests and abilities of the child, as well as social and cultural context all need to be considered for each individual. The complete mathematics alignment document can be found [here](#) in The Alignment of the California Preschool Learning

**COHERENCE FROM PRESCHOOL TO KINDERGARTEN**

**BACKGROUND**

One purpose of California adopting the K-12 Common Core Standards in 2010 was to eliminate the “mile-wide, inch deep” approach to curriculum, which required students to learn many math topics in each grade without much depth of understanding. After 2010, the expectations of Kindergarten students shifted to learning about fewer topics with a deeper understanding of each topic. Since the California Preschool Foundations for mathematics were written before 2010, parts of these foundations still reflect some topics that have now been delayed to future grades or eliminated altogether. The purpose of the recommendations below is to provide educators information about topics to delay or discontinue in preschool and TK in order to ensure a smooth bridge between mathematics in preschool and the elementary grades.
“Instructional time should focus on two critical mathematical areas. One area is representing, relating, and operating on whole numbers . . . The second important area is geometry with a focus on identifying and describing shapes and space; and analyzing, comparing, creating, and comparing shapes. These two areas are intricate and complex and build the foundation for future learning in mathematics. While both prepare the young learner for more formal mathematics instruction, learning time should be devoted to number sense more than any other topic in mathematics.” —CDE 2011b, 26 (TK chapter from 2013 California Mathematics Framework)

**EARLY MATH DOMAINS**

* **Counting and Operations**
  * **Rationale:** Counting collections of objects and representing math stories with objects are valuable experiences for preschool and TK students. In the California Common Core Standards for Mathematics, fluency for addition and subtraction within 5 is not expected until the end of Kindergarten, and fluency within 10 is not expected until the end of 1st grade. Therefore, the emphasis at this stage of inquiry should be on exploring and representing situations involving operations rather than formal symbolic use.

<table>
<thead>
<tr>
<th>What to Emphasize:</th>
<th>What to Delay until K-2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>★ Child directed counting of objects</td>
<td>★ Symbolic notation of expressions and equations (Ex. 2+3=5)</td>
</tr>
<tr>
<td>★ Child generated stories derived from play using concrete objects to represent story characters/elements</td>
<td>★ Phrases that label operations (Ex. “This story is an addition problem”)</td>
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<tr>
<td>★ Begin to represent and solve stories/situations derived from child directed counting and play</td>
<td>★ Overemphasis on the naming and writing of numerals vs. exploring quantities</td>
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<tr>
<td>★ Adults posing “What if?” questions around combining and reducing groups and objects</td>
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</tbody>
</table>

1 "The Young Child & Mathematics - Third Edition" by Angela Chan Turrou, Nicholas C. Johnson, and Megan L. Franke (pgs. 9-27) provides guidance about Counting Collections.

2 Developing Number Concepts: Books 1 and 2 by Kathy Richardson provides information about using counting boards to create stories.

3 The Kindergarten chapter of the 2013 California Mathematics Framework advocates for a gradual introduction of symbols in Kindergarten.

DREME resources for **Counting** and **Operations**
**Spatial Relations**
- **Rationale:** Students developing ideas around spatial relations is more complex than we often assume. Teachers can help extend and make explicit students’ ideas about spatial relations through activities and a variety of play-based approaches. Spatial relations are crucial to later understanding of geometric and algebraic concepts. The California Common Core Standards for Mathematics emphasize student learning of attributes, while students develop formal language for shapes K-5.

<table>
<thead>
<tr>
<th>What to Emphasize:</th>
<th>What to Delay until K-2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>★ Students describe objects using spatial language</td>
<td>★ Memorization of formal names of shapes</td>
</tr>
<tr>
<td>★ Teacher offers suggestions of spatial language</td>
<td>★ Formal language related to attributes of geometric figures</td>
</tr>
<tr>
<td>★ Student-directed building of shapes from smaller shapes²</td>
<td>(vertical line, right angle)</td>
</tr>
<tr>
<td>★ Student-directed taking apart shapes³</td>
<td></td>
</tr>
<tr>
<td>★ Students communicating about and describing shapes and attributes of shapes</td>
<td></td>
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<tr>
<td>using informal language and gestures (“that shape isn’t finished”, “that shape</td>
<td></td>
</tr>
<tr>
<td>has a sharp point”)</td>
<td></td>
</tr>
</tbody>
</table>

1 "The Young Child & Mathematics - Third Edition" by Angela Chan Turrou, Nicholas C. Johnson, and Megan L. Franke (pgs. 29-43) provides guidance about strategies to encourage spatial language.

2 The Kindergarten chapter of the 2013 California Mathematics Framework outlines expectations for spatial language development.

**DREME resources for Spatial Relations**

**Measurement and Data**
- **Rationale:** Students need experiences collecting and organizing data and opportunities to describe their world in terms of measurement (big, small, tall, heavy, high, etc.¹) In the California Common Core Standards for Mathematics, students will use non-standard units of measurement in Kindergarten and 1st grade, and will measure using standard units in 2nd grade. These expectations allow students in pre-K to explore measurement without expectation of becoming proficient using formalized units of measurement (inches, centimeters, feet, etc.) in a pre-K environment.

1 1. First Steps to Math Success: Guidance for Teaching Mathematics in Transitional Kindergarten, June 2023
### What to Emphasize:

- Students use descriptive language to:
  - compare and order 3 or more objects
  - explore length and height
  - explore capacity (volume) and time
- Students collect data around topics they are interested in
- Explore data collection through variation and frequency (e.g. how often people use the playground slides, go to the park or grocery store)
- Students using language to describe how they sort and classify a variety of objects

### What to Delay until K-2:

- Expected proficiency measuring objects using standardized units (inches, feet, etc.)
- Measuring area with standard measuring tools such as a ruler
- Telling time

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1 "The Young Child & Mathematics - Third Edition" by Angela Chan Turrou, Nicholas C. Johnson, and Megan L. Franke (pgs. 47-69) provides guidance about strategies to explore measurement and data.

**DREME resources for Measurement and Data**

- **Patterns and Algebra**
  - **Rationale:** Working with patterns and learning to look for, describe and extend patterns are essential precursors to algebraic reasoning. According to the California Common Core Standards for Mathematics, labeling patterns (AB, ABBA, etc.) is not part of future mathematics work in elementary grades.

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### What to Emphasize:

- Students create patterns from blocks or other toys/tools
- Students generalize patterns in their surroundings. Students recognize patterns and in the creations of others
- Students reproduce patterns
- Students extend patterns
- Students use language to describe how they sort and classify a variety of objects

### What to Delay until K-2:

- Eliminate naming or labeling patterns (as in AB, ABBA, etc.) in favor of letting students describe what the pattern is and how it works

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First Steps to Math Success: Guidance for Teaching Mathematics in Transitional Kindergarten, June 2023
"The Young Child & Mathematics - Third Edition" by Angela Chan Turrou, Nicholas C. Johnson, and Megan L. Franke (pgs. 71-85) provides guidance on how to encourage productive pattern work.

DREME resources for Patterns and Algebra

PLAY

Play is an essential element of healthy development for children of all ages. Play influences all areas of development, offering children the opportunity to learn about self, others, and the physical environment. Play encourages aspects of social, emotional, cognitive, and physical development. Children learn how to interact with peers when engaged in play activities while also building on important schemas about the real world.

According to psychologist Jerome Bruner (1972): Play appears to serve several centrally important functions. First, it is a means of minimizing the consequences of one’s actions and of learning, therefore . . . [it is] . . . a less risky situation . . . Second, play provides an excellent opportunity to try combinations of behavior that would, under functional pressure, never be tried. (p. 693) According to Bruner,

Play can be seen as a way for children to take risks without fear of failure. 

His definition also proposes that creativity and play activities are closely related. That is, if children explore and experiment in their play, the possibilities for creative outcomes are greatly enhanced without the fear of failure. (Implementing the Early Childhood Curriculum Part IV- 266)

Social-emotional skills emerge through children’s experience in close relationships and in the varied activities that occur in relational experience, such as shared conversation, warm nurturance, and guided assistance in learning capacities for sociability, responsibility, and self-control. It is apparent that many kinds of play contribute to social-emotional competence in preschoolers, including social play with caregivers and peers, play with toys and other objects, structured group activities, and even games with rules. One conclusion to be derived from this observation is that play is an essential cornerstone of healthy social and emotional development in early childhood and contributes to the skills necessary for adjustment to and success in school. (PSF_Vol.1, SED p.4)

Play has many benefits that correlate specifically with mathematics. Through play with objects, such as blocks, sand, balls, crayons, and paper, children practice mathematical skills such as measurement, quantification, classification, counting, ordering, and part-whole relations (Gelfer & Perkins, 1988; Ginsberg, Inoue & Seo, 1999; Piaget, 1962; Ness & Farenga, 2007). Object play, including play with art materials, has been shown to be related to better visuo-spatial skills (Caldera et al., 1999; Hirsch, 1996). Wolfgang, Stannard, & Jones (1996) followed a group of 37 children over 16 years and found that the complexity of their play with blocks as 4-year-olds was significantly and positively related to their level of achievement in mathematics during middle and high school, even controlling for IQ and gender. This research suggests that complex object play could provide children with the early mathematical understanding that supports later learning in formal contexts.
Provocations allow and encourage children to experience the world for themselves through open-ended activities without being overtly guided by an adult. Teachers provide materials, media, and general direction, as needed, but the children take the ideas where they want. This allows children to develop skills of creativity, inventiveness and flexibility in thinking and planning.

PLAY IS ESSENTIAL

The CA Department of Education published *The Powerful Role of Play in Early Education*, which states, “When play is at the center of the curriculum in high-quality play-based programs, teachers plan for and allow children to participate in several types of play across this continuum (See Figure 1).” Each of the three points across the continuum are designed with the intention to elicit different experiences for early learners.

FIGURE 1. PLAY-BASED LEARNING ACROSS A CONTINUUM

CHILD SELF-DETERMINED PLAY is centered and driven by the child’s interests. During this time, children have the freedom and agency to choose their activities and materials to play with. They decide what they want to explore, how and with whom (if any) do they engage with during this “free play” time, and what the direction and outcomes may be during this experience. A teacher’s role is to observe, listen, and acknowledge children during this type of play.

ADULT-CHILD COLLABORATIVE PLAY is driven by children’s interests with adults interacting in meaningful ways to help guide learning and strengthen skills and knowledge. Both may work collaboratively to create the play context, identify use of materials and play partners, and determine the purpose and outcomes for the play. While there is a collective responsibility and ownership of learning, teachers may offer more guidance and support based on the context as well as the students’ needs.
ADULT PLANNED AND DIRECTED PLAY is intentionally designed and initiated by the adult with children engaged in the adult directed activity. With the adult as the lead, directions are provided and learning goals are established giving students practice with concepts, skills, and dispositions that enhance understanding and learning. The adult may use explicit instruction and scaffolds to support children in achieving the learning goal. For instance, a teacher may use an animal theme unit being studied as content when directing learning around classification. The teacher may provide children with toy animals to sort and classify by color, size, shape, number of legs, farm animals vs. non-farm animals, pets vs. wild animals, etc. As students sort by attribute, the CA Preschool Learning Foundations, Volume 1 states, “Analyzing, comparing, and classifying objects provide a foundation for algebraic thinking.”

Recommended video: Teaching Maths Through Play in Preschool https://youtu.be/yOlj5ctEorI
For additional vignettes and examples of student talk and play as it relates to mathematics domains, see the CA Preschool Curriculum Framework, Volume 1, Chapter 6 and the CA Preschool Learning Foundations, Volume 1, Foundations in Mathematics section.

CLOSING
Universal PreKindergarten will bring fresh ideas into education, and challenges to be embraced. Supporting the learning of our youngest students can be a joyful experience with rewards for years to come. The County Office system is here to support and encourage all districts as they move forward to make their UPK programs worthwhile and supportive.

NCTM states
Mathematics learning at this level must be active, rich in natural and mathematical language, and filled with thought-provoking opportunities. Students respond to the challenge of high expectations and mathematics should be taught for understanding rather than around preconceptions around children’s limitations. This does not mean abandoning children’s ways of knowing and representing; rather it is a clear call to create opportunities for young students to learn new, important mathematics in ways that make sense to them.

NCTM Principles and Standards for School Mathematics, 2000, p. 77
References


2013 California Mathematics Framework: Kindergarten grade level chapter